GASpAR Development

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1 Historical and scientific context

Accurate and efficient simulation of strongly turbulent flows is a prevalent challenge in many atmospheric, oceanic, and astrophysical applications. New simulation codes are needed to investigate such flows in the parameter regimes that interest the geophysics communities. Turbulent flows are linked to many issues in the geosciences, for example, in meteorology, oceanography, climatology, ecology, solar-terrestrial interactions, and solar fusion, as well as dynamo effects, specifically, magnetic-field generation in cosmic bodies by turbulent motions. Nonlinearities prevail when the Reynolds number Re (the ratio of the nonlinear to diffusive terms in the Navier–Stokes equations) is large. The number of 3-dimensional degrees of freedom (d.o.f.) increases as $\mathsf{Re}^{9/4}$ as $\mathsf{Re} \to \infty$ in the Kolmogorov 1941 framework (7, §7.4) . For aeronautic flows often $\mathsf{Re} > 10^6$, but for geophysical flows often $\mathsf{Re} \gg 10^8$ (3; 11)

Computations of turbulent flows must contain enough scales to encompass the energy-containing and dissipative scale ranges *distinctly*. Uniform-grid convergence studies on 3D compressible-flow simulations show that in order to achieve the desired scale content, uniform grids must contain at least 2048^3 cells (15). Today such computations can barely be accomplished. A pseudo-spectral Navier-Stokes code on a grid of 4096^3 uniformly spaced points has been run on the Earth Simulator (8), but the Taylor Reynolds number ($\propto \sqrt{\mathsf{Re}}$) is still no more than ≈ 700, very far from what is required for most geophysical flows. The *main goal of our efforts* is to ask, if the significant structures of the flow are indeed sparse, so that their dynamics can be followed accurately even if they are embedded in random noise, then does dynamic adaptivity offer a means for achieving otherwise unattainable large Re values.

Thus, we have undertaken a long-term development program to provide a dynamic geophysical and astrophysical spectral-element adaptive refinement (GASpAR) code for simulating and studying turbulent phenomena.

Several properties of spectral-element methods (SEMs, 1; 13) make them desirable for direct numerical simulation of geophysical turbulence. Perhaps most significant is the fact that SEMs performed at high polynomial degree are inherently minimally diffusive and dispersive. The extent of the spatial and temporal scales that characterize turbulence depends critically on Re, so to draw conclusions we must be certain of this number in our computations. Thus, we cannot allow the numerical methods themselves to introduce diffusion. Also, because SEMs use finite elements, they can be used efficiently in high-resolution turbulence studies in domains with complicated boundaries. These qualities, together with their good scalability properties (e.g., 6), suggest spectral element methods to be a good basis for high-order adaptive modeling of turbulent flows.

2 How this work supports NCAR strategic priorities

This development work relates directly to at least two of NCAR's strategic priorities: (1) Conducting research in computer science, applied mathematics, statistics, and numerical methods; and (2) Developing and providing advanced services and tools. The connection to both priorities is quite clear, as we are indeed trying to establish the ability of these new high-order adaptive methods to model turbulence, and this has necessitated a good deal of innovation in applied mathematics and numerical methods. Furthermore, the code was released in December 2005, in direct support of priority (2), and, periodic updates are forthcoming. See `http://www.cisl.ucar.edu/nar/2006/4i.6.nt.jsp` and `http://www.cisl.ucar.edu/nar/2006/4i.5.ts.jsp` for additional details.

3 FY2007 accomplishments

FY2007 has seen that the plans outlined in the FY2006 Annual Report were largely met. We completed the development and testing of an explicit incompressible MHD solver. This solver uses the Elsasser (2) formulation of the MHD equations, and used a Runge-Kutta (RK) algorithm for time stepping. Each RK stage requires that the divergence constraints for velocity and magnetic field be maintained, necessitating linear solves for the pressure for each Elsasser equation. Our results (14) indicate the method compares extremely well with the pseudo–spectral method with regard to numerical accuracy, on a rather difficult problem in the literature. We also were able to demonstrate the effect of low order polynomials on sup-norm values, showing that local low order may prove detrimental in predicting important quantities in MHD flows such as magnetic reconnection rates, despite nominally high resolution of the simulation.

In addition, we made progress in applying this solver to another important problem related to coronal heating: the magnetic island instability. In this study we compare the results of the adaptive spectral element code with both a pseudo–spectral method, and with a low–order adaptive finite difference (FD) code. We see from the preliminary results that local low order behaves somewhat differently from the above sup-norm problem in that the most striking difficiency is its inability to maintain energy conservation to a sufficient level for long-time intergrations of the unstable flow. There are also significant discrepancies in the sup-norm diagnostics as well. The spectral element method, on the other hand, is shown to compare extremely well with the pseudo–spectral calculations.

In the FY2006 report we stated that *we may in FY2007 add to our PDE solver "toolbox" code* *to accommodate the compressible MHD equations, depending on the physics of the objects we want to model*. We have begun development of this solver, but have not yet begun to test it, as there are a number of numerical issues that must still be resolved.

We made a strategic decision not to implement the 'alpha'-model for high Reynolds number calculations in two space dimensions (2D), and instead began code modifications required to carry out simulations in three dimensions (3D). At this point, the subset of the code that computes two dimensional solutions has been tested, and found to agree with the previous full 2D code.

Together with our long term visitor as the lead, we implemented a robust, low memory, third order accurate Runge-Kutta algorithm to replace an algorithm often cited in the literature, but which we show to be only second order for nominal orders greater than 2. This algorithm has been applied to the generalized advection–diffusion solver in the GASpAR code, and shown to behave as expected for explicit time solutions.

Finally, working with the IMAGe Computational Mathematics Group we have made significant strides toward developing an optimized additive Schwarz preconditioner for our iterative Krylov methods in the Navier–Stokes and MHD solvers. Thus far, we have the first stage of the algorithm with nonzero overlap working for nonconforming element descritizations, and are beginning the second stage of optimizing the preconditioner so that the iteration count is bounded.

4 2007 plans

In FY2008, we will complete the development of the 3D code with regular conforming discretizations. To complete the preliminary effort, we must still develop a generic element class that conserves memory better than the 2D code allowed. To effect this, we may also restrict the user to using isotropic polynomial degree. It is possible that we will also undertake the development required to handle nonconforming 3D element connnectivity, and also to accommodate 3D adaptivity. We will continue our development of the optimized preconditioners described briefly above. If we can achieve optimization for conforming elements, we will develop the capability to handle nonconforming elements; this will, in turn, allow us to apply the preconditioner to adaptively refined grids.

5 References

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